

On the Vacuum Cherenkov Radiation in Noncommutative Electrodynamics and the Elusive Effects of Lorentz Violation

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Abstract

We show that in the framework of noncommutative classical electrodynamics Cherenkov radiation is permitted in vacuum and we explicitly compute its spectrum at first order in the noncommutative parameter. We discuss the phenomenological impact of the merge of this new analysis with the old results of the substantial modification to the spectrum of the synchrotron radiation obtained in [1]. We propose to consider the pulsars' radiation spectrum - due to its very strong magnetic field - to investigate these Lorentz violating effects in astrophysical phenomena.

1 Introduction

The possible occurrence of noncommutative spatiotemporal coordinates has been extensively investigated in physics [2] and mathematics [3]. Noncommutative perturbative *quantum* field theories seem unappealing for phenomenology (see, e.g. [4]) – and nonperturbative *quantum* frameworks are just on their way [5] – hence, as in [1], we consider the well behaved noncommutative *classical* electrodynamics (NCED) proposed in [6]. We shall take the view that a nonzero θ – the scale of noncommutativity – breaks Lorentz invariance (c.f. e.g. [7]), hence θ “measures” the amount of Lorentz violation.

In [1] large departures from the ordinary spectrum of the synchrotron radiation were shown for the first time. They are due to the acausal (Lorentz violating) behavior of the electromagnetic fields related to the modified (shifted) poles of the associated Green functions (see [1], [8], and Eqs. (15) and (16) below). Those results pointed out the relevance of strong magnetic fields of astrophysical origin to test Lorentz violation.

Here we want to pursue further that investigation. We do so motivated by the recent result that in a Maxwell-Chern-Simons electrodynamics there is Cherenkov radiation in vacuum [9]. We shall show that this also happens in NCED, explicitly compute the spectrum of this radiation, and investigate whether this novel feature might be useful to improve the bound on θ by astrophysical limits on the Cherenkov radiation.

Noncommutativity for us is expressed in the canonical form [10], $x^\mu * x^\nu - x^\nu * x^\mu \equiv [x^\mu, x^\nu]_* = i\theta^{\mu\nu}$, where the $*$ -product for $\phi(x)$ and $\chi(x)$ is $(\phi * \chi)(x) \equiv \exp\{\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x\partial_\nu^y\}\phi(x)\chi(y)|_{y\rightarrow x}$, $\theta^{\mu\nu}$ is c -number valued, the Greek indices run from 0 to 3. The NCED action with a coupling to an external current J_μ is

$$\hat{I} = -\frac{1}{4} \int d^4x \hat{F}^{\mu\nu} \hat{F}_{\mu\nu} + J_\mu \hat{A}^\mu, \quad (1)$$

where $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - i[\hat{A}_\mu, \hat{A}_\nu]_*$, \hat{A}_μ can be expressed in terms of a U(1) gauge field A_μ and of $\theta^{\mu\nu}$ by means of the Seiberg-Witten (SW) map¹ $\hat{A}_\mu(A, \theta)$. The action (1) at $O(\theta)$ becomes

$$\hat{I} = -\frac{1}{4} \int d^4x [F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} \theta^{\alpha\beta} F_{\alpha\beta} F^{\mu\nu} F_{\mu\nu} + 2\theta^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu} F^{\mu\nu}] + J_\mu \hat{A}^\mu, \quad (2)$$

where we made use of the $O(\theta)$ SW map $\hat{A}_\mu(A, \theta) = A_\mu - (1/2) \theta^{\alpha\beta} A_\alpha (\partial_\beta A_\mu + F_{\beta\mu})$, and of the $*$ -product. Our analysis is based on the theory (2).

We consider processes with emission of electromagnetic radiation by external sources both in vacuum and in presence of an isotropic medium with an electric permeability $\epsilon(\omega)$, that is a function of the frequency of the radiation (the magnetic permeability is taken to be 1). In this case the *linearized* constitutive relations of [6] among the fields in the medium, descending from (2), are only slightly modified and are given by

$$D^i = \varepsilon^{ij}(\omega) E^j \quad \text{and} \quad H^i = (\mu^{-1})^{ij} B^j, \quad (3)$$

where we choose $\theta^{0i} = 0$, $\theta^{ij} = \epsilon^{ijk} \theta^k$, ϵ^{ijk} is the completely antisymmetric symbol, $\varepsilon^{ij}(\omega) = \epsilon(\omega) \xi^{ij}$,

$$\xi^{ij} \equiv a\delta^{ij} + \theta^i b^j + \theta^j b^i, \quad (\mu^{-1})^{ij} \equiv a\delta^{ij} - (\theta^i b^j + \theta^j b^i), \quad (4)$$

¹As $\theta^{\mu\nu} \rightarrow 0$, $\hat{A}_\mu(A, \theta) \rightarrow A_\mu$ and $\hat{F}_{\mu\nu} \rightarrow F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, hence this theory reduces to the ordinary Maxwell theory, as requested.

$a = (1 - \vec{\theta} \cdot \vec{b})$, \vec{b} is the background magnetic field, and the Latin indices run from 1 to 3. The Bianchi identities still hold unmodified, hence $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$ or

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \text{and} \quad \vec{E} = -\frac{1}{c} \frac{\partial}{\partial t} \vec{A} - \vec{\nabla} \Phi. \quad (5)$$

The dynamical Maxwell equations are $\partial_\mu \Pi^{\mu\nu} = J^\nu + \theta^{\alpha\nu} J^\sigma \partial_\alpha A_\sigma + \theta^{\alpha\sigma} \partial_\sigma (A_\alpha J^\nu)$, where $\Pi^{\mu\nu} = \delta \hat{I} / \delta (\partial_\mu A_\nu)$, and they lead to

$$\vec{\nabla} \cdot \vec{D} = 4\pi[\rho + \vec{\theta} \cdot (\vec{\nabla} \times (\rho \vec{A}))], \quad (6)$$

$$\left(\vec{\nabla} \times \vec{H} - \frac{1}{c} \frac{\partial}{\partial t} \vec{D} \right)^i = \frac{4\pi}{c} [J^i + \theta^j (\epsilon^{ijk} J^\sigma \partial_k A_\sigma + \epsilon^{jlk} \partial_k (A^l J^i))] . \quad (7)$$

By using the potentials (5), and the generalized Lorentz gauge $\frac{\epsilon(\omega)}{c} \frac{\partial}{\partial t} \Phi + \vec{\nabla} \cdot \vec{A} = 0$, the Euler-Lagrange equations (6) and (7) become

$$a \square_\epsilon \Phi + (\theta^i b^j + \theta^j b^i) \left[\partial_i \partial_j \Phi + \frac{1}{c} \frac{\partial}{\partial t} (\partial_i A_j) \right] = -\frac{4\pi}{\epsilon(\omega)} [\rho + \vec{\theta} \cdot (\vec{\nabla} \times (\rho \vec{A}))], \quad (8)$$

$$\begin{aligned} a \square_\epsilon A^i + (\theta^i b^j + \theta^j b^i) \left[-\frac{\epsilon(\omega)}{c^2} \frac{\partial^2}{\partial t^2} A_j + \partial_j (\vec{\nabla} \cdot \vec{A}) \right] + \epsilon^{ikm} (\theta^m b^j + \theta^j b^m) \epsilon^{jlp} \partial_k \partial_l A_p \\ = -\frac{4\pi}{c} [J^i + \theta^j (\epsilon^{ijk} J^\sigma \partial_k A_\sigma + \epsilon^{jlk} \partial_k (A^l J^i))] , \end{aligned} \quad (9)$$

where $\square_\epsilon \equiv -\epsilon(\omega) c^{-2} \partial_t^2 + \partial_{x_1}^2 + \partial_{x_2}^2 + \partial_{x_3}^2$, respectively.

In the same spirit of [1], we choose the simplest possible settings to observe the effects of Lorentz violation given by $\vec{b} = (0, 0, b)$ – background magnetic field speeding up the particle; $\vec{\theta} = (0, 0, \theta)$ – i.e. θ^3 is the only nonzero component of $\theta^{\mu\nu}$. Although this choice for $\vec{\theta}$ surely is not the most general, it has the double advantage for us of (a) capturing the essential features of noncommutativity (see, for instance, the acausal behavior of the electromagnetic field [1]) and (b) highly simplifying the Maxwell equations (8) and (9). From these settings: $\xi^{ij} = a \delta^{ij} + \delta^{i3} \delta^{j3} \lambda$, where

$$\lambda \equiv 2 \theta b , \quad (10)$$

$a = (1 - \theta b) = (1 - \lambda/2)$. We are interested only in the largest departures from the ordinary energy spectrum. These are $O(\theta) \cdot O(e^2)$ in the energy – where e is the electric charge – hence contributions higher than $O(e)$ in the fields will be neglected. Taking into account these approximations – and the further suppression at a large distance R from the source, by a factor $1/R$ – Eqs. (8) and (9) can be written as²

$$\square_\epsilon A_1 + \lambda \partial_2 (\partial_1 A_2 - \partial_2 A_1) = -\frac{4\pi}{c} \tilde{J}_1 , \quad (11)$$

$$\square_\epsilon A_2 + \lambda \partial_1 (\partial_2 A_1 - \partial_1 A_2) = -\frac{4\pi}{c} \tilde{J}_2 , \quad (12)$$

$$\square_\epsilon A_3 - \frac{\lambda \epsilon(\omega)}{c^2} \partial_t^2 A_3 - \frac{\epsilon(\omega)}{c} \lambda \partial_3 \partial_t \Phi = -\frac{4\pi}{c} \tilde{J}_3 , \quad (13)$$

$$\square_\epsilon \Phi + \lambda (\partial_3^2 \Phi + \frac{1}{c} \partial_3 \partial_t A_3) = -\frac{4\pi \tilde{\rho}}{\epsilon(\omega)} , \quad (14)$$

²For a more detailed analysis of the approximations leading to Eqs. (11)-(14) c.f. [1].

where $\tilde{J}_i \equiv J_i/a$, $i = 1, 2$, and $\tilde{\rho} \equiv \rho/a$. In the following Section we shall present the computation of the vacuum Cherenkov radiation in these settings, while the last Section is devoted to the discussion of some astrophysical implications and to our conclusions.

2 Vacuum Cherenkov radiation in NCED

In our settings $\vec{\theta}$ only has a z -component, hence the largest modifications of the Cherenkov spectrum are expected when the charged particle moves along the same axis: $\vec{J} = (0, 0, \tilde{J}_3)$. As Eqs. (11) and (12) represent the propagation of plane waves in NCED discussed in [6], the equations of interest for the present analysis are Eqs. (13) and (14) that, in momentum space, have the following solutions

$$A_3(\vec{k}, \omega) = -\frac{4\pi}{c} \frac{\tilde{J}_3(\vec{k}, \omega)}{(1+\lambda)\epsilon(\omega)\frac{\omega^2}{c^2} - \vec{k}^2} - \frac{4\lambda\pi}{c} \frac{\tilde{\rho}k_3\omega}{[(1+\lambda)\epsilon(\omega)\frac{\omega^2}{c^2} - \vec{k}^2](\epsilon(\omega)\frac{\omega^2}{c^2} - \vec{k}^2 - \lambda k_3^2)}, \quad (15)$$

$$\Phi(\vec{k}, \omega) = -\frac{4\pi\tilde{\rho}(\vec{k}, \omega)/\epsilon(\omega)}{\epsilon(\omega)\frac{\omega^2}{c^2} - \vec{k}^2 - \lambda k_3^2} + \frac{4\lambda\pi}{c^2} \frac{k_3\omega\tilde{J}_3}{[(1+\lambda)\epsilon(\omega)\frac{\omega^2}{c^2} - \vec{k}^2](\epsilon(\omega)\frac{\omega^2}{c^2} - \vec{k}^2 - \lambda k_3^2)}, \quad (16)$$

where $\tilde{\rho}(\vec{k}, \omega) = e/(2\pi a) \delta(\omega - k_3 v)$, $\tilde{J}_3(\vec{k}, \omega) = (0, 0, v\tilde{\rho})$ and v is the speed of the particle.

If one defines

$$\vec{E}(\omega) = \frac{1}{(2\pi)^{3/2}} \int d^3k \vec{E}(\vec{k}, \omega) e^{i\vec{w} \cdot \vec{k}} \quad (17)$$

where w is the perpendicular distance from the path of the particle moving along the z -axis and the observation point of $\vec{E}(\omega)$ has coordinates $(0, w, 0)$, the energy per unit distance lost in collisions with the medium with impact parameter $w \geq d$ – where d is a large distance from the path – is seen to be given by [11]

$$\frac{d\mathcal{E}}{dz} = -\text{Re} \left(i \int_0^\infty dw \, w \epsilon(\omega) \mathcal{E}(\omega, \lambda, d) \right), \quad (18)$$

where

$$\mathcal{E}(\omega, \lambda, d) \equiv \int_d^\infty dw \, w |\vec{E}(\omega)|^2, \quad (19)$$

and we wrote explicitly the λ -dependance. For us the relevant components of the electric field are

$$E_2(\omega) = \left(\frac{2}{\pi} \right)^{1/2} \frac{e}{v(1+\epsilon(\omega)\beta^2)} \left[\frac{\sigma_+ K_1(w\sigma_+)}{\epsilon(\omega)} + \beta^2 \sigma_- K_1(w\sigma_-) \right], \quad (20)$$

and

$$E_3(\omega) = \left(\frac{2}{\pi} \right)^{1/2} \frac{-ie\omega(1-\epsilon(\omega)\beta^2)}{v^2(1+\epsilon(\omega)\beta^2)} \left[\frac{K_0(w\sigma_+)}{\epsilon(\omega)} + \beta^2 K_0(w\sigma_-) \right], \quad (21)$$

where $\beta = v/c$,

$$\sigma_+^2 = \frac{\omega^2}{v^2} [1 + \lambda - \epsilon(\omega)\beta^2], \quad \sigma_-^2 = \frac{\omega^2}{v^2} [1 - (1 + \lambda)\epsilon(\omega)\beta^2], \quad (22)$$

and K_n are the modified Bessel functions. Inserting the expressions for the fields (20) and (21) into Eq. (19), we see that for large d , one can use the asymptotic behavior of K_n to write

$$\mathcal{E}(\omega, \lambda, d) = \frac{e^2}{v^2} \left\{ \frac{|A|^2 + |C|^2 \omega^2 / v^2}{2 |\sigma_+| \text{Re}(\sigma_+)} \exp(-2d \text{Re}(\sigma_+)) + \frac{|B|^2 + |D|^2 \omega^2 / v^2}{2 |\sigma_-| \text{Re}(\sigma_-)} \exp(-2d \text{Re}(\sigma_-)) \right\}$$

$$+ 2\text{Re} \left(\frac{AB^* + CD^*\omega^2/v^2}{\sigma_+^2 - \sigma_-^{*2}} \frac{\sigma_+ - \sigma_-^*}{\sqrt{\sigma_+ \sigma_-^*}} \exp(-d(\sigma_+ + \sigma_-^*)) \right) \Bigg\} , \quad (23)$$

where

$$A = \frac{\sigma_+}{\epsilon(\omega)(1 + \epsilon(\omega)\beta^2)} , \quad B = \frac{\beta^2 \sigma_-}{1 + \epsilon(\omega)\beta^2} , \quad C = \frac{1 - \epsilon(\omega)\beta^2}{\epsilon(\omega)(1 + \epsilon(\omega)\beta^2)} , \quad D = \frac{\beta^2(1 - \epsilon(\omega)\beta^2)}{1 + \epsilon(\omega)\beta^2} . \quad (24)$$

For $\lambda = 0$ we have $\sigma_+ = \sigma_- \equiv \sigma$, thus the correct Maxwell limit is recovered because, for large d , $\mathcal{E}(\omega, 0, d) \sim \exp(-2d \text{Re}(\sigma))$, hence the Cherenkov radiation is observed only if $\text{Re}(\sigma) = 0$, which never happens in vacuum - *in a medium* this is the usual condition that gives $v > c$ (c.f. either of Eqs.(22) with $\lambda = 0$).

In NCED, instead, Cherenkov radiation is also allowed *in vacuum*. Indeed, from Eqs. (22) we can have either $\text{Re}(\sigma_+) = 0$ or $\text{Re}(\sigma_-) = 0$, for

$$\lambda < \beta^2 - 1 \quad \text{and} \quad \lambda < 0 , \quad (25)$$

or

$$\lambda > \frac{1 - \beta^2}{\beta^2} \quad \text{and} \quad \lambda > 0 , \quad (26)$$

respectively, where $\lambda = 2\theta b$, hence the dumping at large d in Eq.(23) could, in principle, disappear³. It makes sense to investigate only the behavior in the ultrarelativistic regime: $\beta^2 = 1 - \eta$, where η is a small positive number, by keeping only linear terms in η . To this order both conditions (25) and (26) amount to $|\lambda| > \eta$. We can then pick one of them, say (26), to write the energy loss in a NCED vacuum given in Eq.(18) as⁴

$$\frac{d\mathcal{E}}{dz} = \int_0^\infty d\omega \, \omega \frac{(v^2/\omega^2)|B|^2 + |D|^2}{(1 + \lambda)\beta^2} e^{-2d \text{Re}(\sigma_-)} = \int_0^\infty d\omega \frac{\omega e^2 \beta^2}{c^2} \frac{\lambda - 1 + \beta^2}{(1 + \beta^2)^2} , \quad (27)$$

where we explicitly wrote $1 = \exp(-2d \text{Re}(\sigma_-))$ to make clear the connection with Maxwell theory. By neglecting $O(\eta^2)$ and $O(\eta\lambda)$ contributions, Eq. (27) reads

$$\frac{d\mathcal{E}}{dz} \sim \frac{1}{4} \int_0^\infty d\omega \frac{\omega e^2}{c^2} (\lambda - \eta) . \quad (28)$$

The energy loss corresponding to the other condition, i.e. to Eq. (25), in the same approximation, is given again by Eq. (28), but with λ replaced by $|\lambda|$. Finally the same conditions, Eqs. (25) and (26), can be recovered by the expression of the emission angle of the radiation (see [11]).

3 Phenomenological Implications and Conclusions

Let us summarize here our understanding of the state of the art:

³The real part of the third exponential $\exp(-d(\sigma_+ + \sigma_-^*))$ in Eq. (23) can never be 1 for $\lambda \neq 0$.

⁴As customary, we used $\epsilon = 1 + i\text{Im} \, \epsilon$ and only at the end set $\text{Im} \, \epsilon = 0$.

In [1] the departures from the ordinary spectrum of the synchrotron radiation due to Lorentz violating (acausal) behaviors of the electromagnetic fields gave a large correction factor X to the energy emitted by the source⁵

$$X < \left(\frac{\omega_0}{\omega}\right)^{2/3} n \times 10^{-21} \times \left(\frac{\mathcal{E}(\text{MeV})}{\text{MeV}}\right)^4, \quad (29)$$

where this formula holds in the ultra-relativistic approximation for $\omega_0 \ll \omega \ll \omega_c$, ω_0 is the cyclotron frequency ($\omega_c = 3\omega_0\gamma^3$), $\gamma = (1 - \beta^2)^{-1/2}$, n is the value of the background magnetic field b expressed in Tesla, \mathcal{E} is the energy of the source and we used the bound [13] $\theta < 10^{-2}(\text{TeV})^{-2}$. From (29) it was clear that, in spite of the γ^4 correction, only very strong magnetic fields b could improve the current bound on θ . Hence strong magnetic fields of astrophysical origin were seen to be the proper candidates to test Lorentz violation in this framework. The generality of this result resided in the poles-shift mechanism that one has to expect in a Lorentz violating electrodynamics. This point was also addressed in [8].

In [6] it was shown that plane waves in NCED have a deformed dispersion relation $\omega/c = k(1 - \vec{\theta}_T \cdot \vec{b}_T)$. This modifies the kinematical thresholds of processes involving radiation, such as $\gamma\gamma \rightarrow e^+e^-$, and otherwise forbidden decays, such as $\gamma \rightarrow e^+e^-$, are permitted. The astrophysical observation of ultra high energy gamma rays put some tight bounds on the modification of these thresholds in any theory involving violation of Lorentz invariance [14], [15] (for a review see e.g. [16]). In NCED these kinematical arguments do not improve the present bound on θ as the galactic and extragalactic magnetic field is too weak [17]. As already noted in various occasions, however, a sound quantum version of NCED is not available. Thus, the considerations about such radiation processes only refer to the modified mass-shell (or light cone) of the theory.

Let us now consider the possibility of a phenomenological bound on θ coming from the Cherenkov radiation in the NCED vacuum just obtained in the previous Section. The kinematical analysis of vacuum Cherenkov radiation, performed in other Lorentz violating theories of electrodynamics, also introduces strong limits on Planck scale [18]. The 50 TeV gamma rays experimentally seen from the Crab nebula should come from highly energetic electrons which are explained by inverse Compton scattering. But this rules out the vacuum Cherenkov radiation because the Cherenkov rate is orders of magnitude higher than the inverse Compton scattering rate [15], [18].

For us this implies that the conditions in Eqs. (25) and (26) are not fulfilled, which in turn produces a limit on θ . However, even by considering electrons of very high energy (such as 1.5 PeV [18]) with the present bound on θ one obtains the condition $n \cdot 10^{-23} < 10^{-19}$, which is largely satisfied. Thus tight bounds could only be obtained by considering very strong magnetic fields, such as those observed in compact stars. Since the production of high energy gamma rays from e.g. pulsars, is a complex multi-step process involving shock waves, primary and secondary emissions, synchrotron radiation, inverse Compton scattering, and more [19], one should evaluate how and how much each of those steps is modified. Numerical evaluations might be a conceivable approach to this involved analysis.

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⁵Similar large corrections appear to be reobtained in [12] for the synchrotron in Myers-Pospelov effective electrodynamics.

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